Heuristics for Discrete Power Control - A Case-Study in Multi-Carrier DSL Networks

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ABSTRACT

The performance of multi-user digital subscriber line (DSL) networks is limited by the electro-magnetic coupling between twisted pair cables. The adverse effect of this coupling can be reduced by controlling the transmit powers of all lines. The corresponding multi-user, multi-carrier power control problem can be modeled as a multi-dimensional nonlinear Knapsack problem which has previously motivated the application of various mathematical decomposition methods. These methods decompose the problem into a large number of combinatorial per-subcarrier problems. Our main contribution lies in the proposal and analysis of various low-complexity heuristics for these combinatorial problems. We provide insights in the parameter setting as well as simulation results on a large set of 6 and 30-user DSL scenarios. These show that simple randomized greedy heuristics perform well even in case of a very stringent complexity budget and that the heuristics' average suboptimality is dependent on the targeted data-rate.

Keywords: Power Control, DSL, Metaheuristics, Column Generation

1. MOTIVATION

Digital subscriber lines (DSL) are the most widely deployed broadband access technology today, with more than 320 million customers world-wide in 2010 [1]. DSL systems suffer from the electro-magnetic coupling between the twisted pair cables which induces so called "far-end crosstalk" noise at the DSL receivers. This in turn is the main limiting factor for the data-rate performance of current DSL modems. Furthermore, today's DSL systems are based on discrete multi-tone (DMT) modulation which splits the available frequency bandwidth into independent subchannels ("subcarriers"). We consider the problem of optimally controlling the transmitted power levels on each of these subchannels and hence the crosstalk noise as well as the conveyable total data-rate. This problem is also fundamental in multi-carrier wireless networks [2]. The classical objective is the maximization of a weighted sum of data-rates. Recently this technique has also been discovered useful for reducing the system power consumption in DSL [3, 4]. Current state-of-the-art multi-carrier power control algorithms for tens of subscriber lines are based on techniques such as user-iterative power updates, dual relaxation of transmission rate and sum-power constraints, as well as successive continuous and convex approximation, cf. [3, 5, 6] and the references therein. Dual relaxation results in the independent optimization of a large number of per-subcarrier problems. The distinctive feature of the nonlinear Dantzig-Wolfe decomposition [7, Ch. 23] based scheme in [8] is that it allows for the suboptimal solution of the independent per-subcarrier problems.

Our main contribution is the proposal of various heuristics for complexity reduction of solving the combinatorial per-subcarrier optimization subproblems, thereby expanding upon the work in [8]. We begin in Section 2 by reviewing the optimization problem of controlling the transmit power in DSL. In Section 3 we then turn to the main focus of this paper, namely the combinatorial per-subcarrier problems and various heuristics for their solution. Section 4 gives an example of the heuristics' performance when applied in conjunction with the framework in [8] to solve the main problem from Section 2. Our conclusions are summarized in Section 5.

2. BACKGROUND - GLOBAL PROBLEM

We denote the index sets of users and subcarriers by $\mathscr{U} = \{1,\ldots,U\}$ and $\mathscr{C} = \{1,\ldots,C\}$, respectively, where U and C are the total number of users and subcarriers, respectively. The optimization variables are the power levels p_u^c of user u on subcarrier c, where we will compactly write $\mathbf{p}^c \in \mathscr{R}_+^U$ for the power allocation of all users on subcarrier c. The data-rate of user u on subcarrier c is a nonlinear function $r_u^c(\mathbf{p}^c)$ [9] which notably depends on the power allocation of all users on that subcarrier. Again, we will compactly write $\mathbf{r}^c(\mathbf{p}^c) \in \mathscr{R}^U$ to denote all users' rates on subcarrier c. Reversely, the power allocation $\mathbf{p}^c(\mathbf{r}^c)$ for rates $\mathbf{r}^c$ can be computed as the unique [10] solution of a system of linear equations of size $U \times U$. Power levels are constrained by a regulatory power mask constraint $p_u^c \leq \hat{p}_u^c$ and the implicit constraint $r_u^c(\mathbf{p}^c) \in \mathscr{B}, \forall u \in \mathscr{U}, c \in \mathscr{C}$, motivated by practical modulation schemes which only support a discrete set of data-rates $\mathscr{B}$. Altogether we may compactly write the set of feasible power allocations on subcarrier c as

$$\mathscr{Q}^c = \{\mathbf{p}^c | r_u^c(\mathbf{p}^c) \in \mathscr{B},\ 0 \leq p_u^c \leq \hat{p}_u^c, \forall u \in \mathscr{U}\}. \tag{1}$$

Additional to these per-subcarrier constraints the U users have minimum target-rates $\mathbf{R} \in \mathscr{R}_+^U$ dependent on the accepted service level, as well as technology-dependent maximum sum-power levels $\mathbf{P} \in \mathscr{R}_+^U$. Our optimization objective is defined as the sum of per-subcarrier objectives $\bar{f}_c(\mathbf{p}^c, \hat{\mathbf{w}}, \check{\mathbf{w}}), c \in \mathscr{C}$. These are given as the following weighted sum of users' transmit powers and rates

$$\bar{f}_c(\mathbf{p}^c, \hat{\mathbf{w}}, \check{\mathbf{w}}) = \hat{\mathbf{w}}^\intercal \mathbf{p}^c - \check{\mathbf{w}}^\intercal \mathbf{r}^c(\mathbf{p}^c),\ \forall c \in \mathscr{C}, \tag{2}$$

where the weights $\hat{\mathbf{w}}, \check{\mathbf{w}} \in \mathscr{R}_+^U$ allow us to trade-off between rate and energy optimization, i.e., we can consider rate-maximization and energy-minimization as special cases. We are now ready to formally write the optimization problem for multi-user power control in DSL as the following multi-dimensional nonlinear Knap-

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sack problem [11]

$$\underset{\mathbf{p}^c\in\mathscr{Q}^c,\forall c\in\mathscr{C}}{\text{minimize}} \sum_{c\in\mathscr{C}} \bar{f}_c(\mathbf{p}^c,\hat{\mathbf{w}},\check{\mathbf{w}}) \tag{3a}$$

$$\text{subject to} \quad \sum_{c\in\mathscr{C}} \mathbf{r}^c(\mathbf{p}^c) \succeq \mathbf{R}, \tag{3b}$$

$$\sum_{c\in\mathscr{C}} \mathbf{p}^c \preceq \mathbf{P}. \tag{3c}$$

3. THE COMBINATORIAL SUBPROBLEM

3.1. Subproblem Formulation

After Lagrange relaxation of constraints (3b) and (3c), respectively, one faces for each subcarrier $c\in\mathscr{C}$ an independent, non-linear (and non-convex), wide-sense combinatorial [12, Sec. 4.4] pricing subproblem in the form of [8]

$$\underset{\{\mathbf{r}^c|\mathbf{p}^c(\mathbf{r})\in\mathscr{Q}^c\}}{\text{minimize}} f_c(\mathbf{r}^c) = \bar{f}_c(\mathbf{p}^c(\mathbf{r}^c),\hat{\mathbf{w}}+\boldsymbol{\nu},\check{\mathbf{w}}+\boldsymbol{\lambda}), \tag{4}$$

where $\boldsymbol{\nu},\boldsymbol{\lambda}\in\mathscr{R}_+^U$ are the Lagrange multipliers associated with constraints (3b) and (3c), respectively, cf. [8] for details. Note that for reasons of algorithm design we use the discrete vector $\mathbf{r}^c$ as our variables instead of the uniquely coupled power allocation $\mathbf{p}^c$ used above. In the following sections we study the solution of the subproblem in (4) and therefore omit subcarrier indices c for ease of notation. The search space we consider is $\times_{u\in\mathscr{U}}\mathscr{B}$, i.e., we only search over discrete rate allocations and do not explicitly consider the constraints in (4). An allocation $\mathbf{r}$ violating these constraints has by definition an objective $f(\mathbf{r})=\infty$ and our algorithms thereby never traverse infeasible allocations. As mentioned above, in order to evaluate the objective $f(\mathbf{r})$ and to determine feasibility we need to solve a linear system of equations, cf. (1) and (4). Later we will introduce the number of evaluations of $\mathbf{p}(\mathbf{r})$ as a reproducible complexity measure to compare different algorithms.

The optimal solution of the problem in (4) was shown to have polynomial complexity in [8]. However, obtaining optimal solutions for practical values of U was found intractable for conventional branch-and-bound schemes [8, 13]. Furthermore, the number of these per-subcarrier problems is in the order of thousands in the newest generations of DSL technology. This altogether motivates our work on fast heuristics in the following sections.

3.2. Constructive Greedy Base Heuristics

In the full paper we review the greedy base heuristic as well as the sequential greedy heuristic in [8] and provide an analysis of various 6-user VDSL scenarios, cf. Section 3.5 for simulation details. This analysis shows that the suboptimality of the base heuristic is zero for all collocated network scenarios while the highest suboptimality appears in classical near-far type of scenarios. This insight will guide the parameter settings of randomized heuristics below. Basically two approaches will be taken in the following to improve upon purely greedy schemes, namely a) a randomization of greedy decisions, and/or b) randomized local searches.

3.3. Local Search

Local search schemes aim at iteratively *improving* a given solution $\mathbf{r}$. Their key ingredient is the definition of a neighborhood $\mathscr{N}(\mathbf{r})\subseteq\times_{u\in\mathscr{U}}\mathscr{B}$ around $\mathbf{r}$ from which a next candidate allocation is picked, cf. [14] for various examples of local search schemes. Here we restrict ourselves to two possible neighborhood definitions: The first is a simple one-step neighborhood

$$\mathscr{N}^{(1)}(\mathbf{r}) = \{\tilde{\mathbf{r}}\in\times_{u\in\mathscr{U}}\mathscr{B} \mid \tilde{r}_u = r_u\pm\Delta, \\ \tilde{r}_i = r_i, \forall i\in\mathscr{U}\setminus\{u\}, u\in\mathscr{U}\}, \tag{5}$$

which contains all allocations $\tilde{\mathbf{r}}$ that can be reached by perturbing a single element of $\mathbf{r}$ by Δ. The second used neighborhood is

$$\mathscr{N}^{(2)}(\mathbf{r}) = \mathscr{N}^{(1)}(\mathbf{r})\cup\bar{\mathscr{N}}^{(2)}(\mathbf{r}), \tag{6}$$

$$\bar{\mathscr{N}}^{(2)}(\mathbf{r}) = \{\tilde{\mathbf{r}}\in\times_{u\in\mathscr{U}}\mathscr{B} \mid \tilde{r}_u = r_u\pm\Delta, \tilde{r}_{\bar{u}} = r_{\bar{u}}\pm\Delta, \\ \tilde{r}_i = r_i, \forall i\in\mathscr{U}\setminus\{u,\bar{u}\}, u\neq\bar{u}, u,\bar{u}\in\mathscr{U}\}, \tag{7}$$

which contains all allocations $\tilde{\mathbf{r}}$ that can be reached by perturbing at most two different elements of $\mathbf{r}$ by Δ. Furthermore, two neighborhood search strategies are considered, namely the “first-improving” and the “best-improving” search strategy, cf. [14, Ch. 8].

Name	Abbr.	Reference
Joint Greedy Optimization	JOGO	[8]
Sequential Greedy Optimization	SEGO	[8]
Local Search	LS	Section 3.3
Rollout Algorithm	RA	[15]
Greedy Rand. Adapt. Search Proc.	GRASP	[14, Ch. 8]
Iterated Local Search	ILS	[14, Ch. 11]
Simulated Annealing	SA	[14, Ch. 10]
Ant Colony System	ACS	[16]
Randomized SEGO	rSEGO	Section 3.4
Randomized LS	rLS	Section 3.4
Solver “Couenne”	COU	[17]
Optimal Branch-and-Bound	OPT	[8]

Table 1: *Heuristics compared on the problem in (4).*

3.4. Heuristics Inspired by Meta-Heuristics

In the full paper we will present detailed descriptions of various heuristics for the bit-loading problem in (4) which are partly inspired by well-known meta-heuristics, cf. the overview of all studied algorithms in Table 1. Rollout algorithms and rSEGO/ant colony system algorithms are deterministic and randomized sequential decision making algorithms, respectively. GRASP is an extension of the greedy base heuristic using randomization, while iterated local search, randomized local search, as well as simulated annealing are randomized local search schemes.

3.5. Methodology, Simulations and Discussion

In order to be able to compare to optimal schemes as in [8] we restrict ourselves in this section to $U=6$ users. We construct our network scenarios using a set of specified line lengths $\mathscr{L}=\{200,400,600,800\}$ m, considering all U-combinations with repetitions. For example, for $U=6$ this results in

$$m = \binom{|\mathscr{L}|+U-1}{U} = 84, \tag{8}$$

generated network scenarios. Note that this allows us to identify scenarios where the given algorithms perform badly. Such scenarios were used to initially set the algorithmic parameters. Based on these settings various parameter changes were selected and the impact on the average performance studied by Monte-Carlo simulation. As in [8] we use equal Lagrange multipliers λ_u,ν_u, for all $u\in\mathscr{U}$. For setting the parameters of the heuristics we chose Lagrange multipliers $\lambda_u=1,\nu_u=0$ and weights $\check{w}_u=0,\hat{w}_u=1/U$, which leads to a maximum sum-rate in our 6-user scenarios [8].

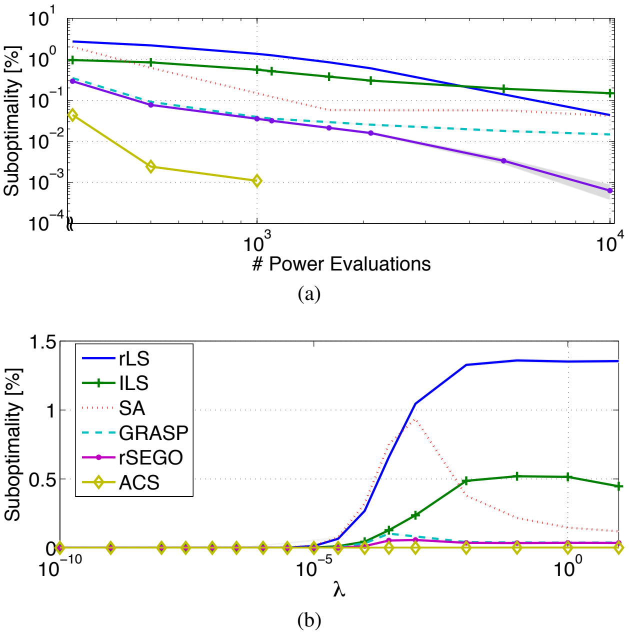

Figure 1: *Average suboptimality of randomized heuristics in 6-user VDSL scenarios; a) Dependency on the complexity budget; b) Dependency on Lagrange multipliers $\lambda_u = \lambda, \forall u \in \mathscr{U}$.*

We further make the practical assumption that there is a restriction in simulation time for solving the subproblems in (4). However, in order to make our results reproducible we will use the number of power evaluations $\mathbf{p}(\mathbf{r})$ by solving a linear system of equations [9] as the stopping criterion of all considered heuristics. Note that this metric also includes evaluations of infeasible allocations, cf. the discussion in Section 3.1. Simulation results provided in the full paper further motivate this complexity metric as it was found to preserve the comparability among different heuristics. While for six users we were able to compute the optimum of the problem in (4), for a higher number of users we either compare to the greedy base heuristic JOGO due to its simplicity, or to a lower bound being the optimal objective of a discrete, convex problem relaxation, cf. [8, Alg. 5] for an analytic solution with $O(U)$ complexity. We find that this lower bound gives a low gap to the optimal objective when the Lagrange multipliers $\boldsymbol{\lambda}$ (and therefore the users' rates) are low. Simulations were carried out using our DSL simulator available in [18] and using common parameters as in [8].

We investigated the solution quality of all presented heuristics for solving the subproblems in (4) in a VDSL system with 1635 subcarriers, where for the comparisons in this section we only select a subset of subcarriers $\tilde{\mathscr{C}} = 1, 51, \ldots, 1601$. As a benchmark for all our algorithms we use "Couenne" [17], a free branch-and-bound based solver for non-convex mixed-integer problems. As a base-line for our stochastic heuristics we added a randomized local search (rLS) scheme where the LS algorithm is reinitialized at random starting points $\mathbf{r}$ uniformly drawn from $\times_{u \in \mathscr{U}} \mathscr{B}$. In the full paper we provide the specific parameter settings and the intuitions behind these settings for all heuristics described in Section 3.4. Figure 1(a) depicts the average suboptimality of all randomized heuristics as a function of the complexity budget in various 6-user VDSL network scenarios. Intuitively, allowing the algorithms to test more solutions leads on average to a better performance. ACS performs best in these test scenarios, where its curve stops at 10^3 as it is optimal on the simulated points beyond that. Note that rLS eventually performs better than ILS and SA, which hints at insufficient diversification capabilities of these two schemes. Figure 1(b) similarly shows, for fixed complexity budget of 10^3 evaluations, the dependency of the heuristics' average suboptimality on

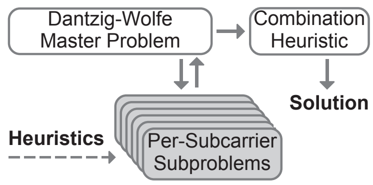

Figure 2: *Framework [8] for applying heuristics in DSL.*

the Lagrange multipliers $\lambda_u = \lambda, \forall u \in \mathscr{U}$, and hence on the targeted transmission rate as the average rate per user increases with these multipliers. JOGO, which is used as an initial incumbent for all schemes, was found to have a monotonously increasing suboptimality with λ. Also, the optimal rates do not change in most scenarios above $\lambda = 10^{-2}$. Differently to JOGO, all heuristics show a peak suboptimality for a specific multiplier value, however, at different values for different heuristics. Intuitively this can be explained by the fact that with increasing λ what matters most is the total number of bits achieved by all users. Then it matters less *how* the bits are distributed among the users as this distribution only influences the power consumption which has a comparably low weight in the objective for high λ. In 30-user VDSL scenarios with a complexity limit of $2 \cdot 10^4$ power evaluations per subcarrier problem and using the same parameter settings for all algorithms as above the picture is very different. The randomized heuristics GRASP, rSEGO and ILS perform now best, with an improvement upon the objective values achieved by the greedy base heuristic by on average 9.9%, 9.8%, and 9.2%, respectively. Note however that the simple *deterministic* extension of the greedy constructive heuristic by a two-step local search improves the greedy heuristic already by on average 8% while taking on average only $0.4 \cdot 10^4$ power evaluations. Notably, the maximum improvement in sum-objective over all 33 tested subcarriers encountered in any tested network scenario was as high as 32%. Further insights in the performance of all heuristics in Table 1 for 6 and 30-user VDSL scenarios will be given in the full version of this paper.

4. PERFORMANCE EVALUATION FOR DSL

The purpose of this section is to provide evidence of the practical usefulness of the proposed approach based on heuristics. As these only target the subproblems in (4) we exemplarily apply the heuristic rSEGO in conjunction with the complexity reduction technique in [19] and the column generation framework in [8], cf. Figure 2. The algorithm consisting of these techniques is compared to state-of-the-art multi-carrier power control algorithms [5, 20]. When using the dual relaxation based, iterative spectrum balancing algorithm (ISB) in [5] we subsequently use the greedy central discrete bit-loading algorithm (CDBL) in [20] to obtain a discrete feasible solution. As an example of the DSL performance we consider a sum-rate maximization problem in a near-far downstream scenario with 50 collocated users, where 40 lines connect to the central office at a distance of 800 m and 10 lines connect to a closer remote cabinet at 200 m distance. Compared to CDBL we obtain a 2.1% sum-rate increase, or more importantly an 8.3% sum-rate increase for the lines connected to the central office. Comparing to ISB our results show an 8.2% increase in total sum-rate and a 12.3% increase in sum-rate for the lines connected to the central office. The full paper will provide simulation settings and extensive average performance and complexity comparisons.

5. CONCLUSIONS

We studied the application of various heuristics to a combinatorial, non-convex power allocation problem in digital subscriber lines (DSL). Parameter setting for various $6-$user DSL scenarios allowed to obtain near-optimal results using several of the proposed randomized heuristics. Under various $30-$user scenarios extending the greedy constructive heuristic by a proposed local search scheme gave already substantial improvements at low complexity. Randomized heuristics still gave slight improvements beyond that for moderate complexity limits. Summarizing, the proposed heuristics have shown an average gain in objective value compared to the greedy constructive heuristic of up to 10%.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

[1] F. Vanier, “World broadband statistics: Short report Q3 2010,” Point Topic Ltd., Tech. Rep., December 2010.

[2] C. Wong, R. Cheng, K. Letaief, and R. Murch, “Multiuser OFDM with adaptive subcarrier, bit, and power allocation,” *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 10, pp. 1747–1758, October 1999.

[3] M. Wolkerstorfer, D. Statovci, and T. Nordström, “Dynamic spectrum management for energy-efficient transmission in DSL,” in *IEEE ICCS 2008*, Guangzhou, China, 19–21 November 2008.

[4] M. Guenach, C. Nuzman, K. Hooghe, J. Maes, and M. Peeters, “Reduced dimensional power optimization using class AB and G line drivers in DSL,” in *IEEE GLOBECOM Workshops*, Miami, USA, 6–10 December 2010, pp. 1443–1447.

[5] W. Yu and R. Lui, “Dual methods for nonconvex spectrum optimization of multicarrier systems,” *IEEE Transactions on Communications*, vol. 54, no. 7, pp. 1310–1322, July 2006.

[6] P. Tsiaflakis, M. Diehl, and M. Moonen, “Distributed spectrum management algorithms for multiuser DSL networks,” *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 4825–4843, October 2008.

[7] G. Dantzig, *Linear programming and extensions*. Princeton University Press, 1963.

[8] M. Wolkerstorfer, J. Jaldén, and T. Nordström, “Complexity reduction strategies for the discrete multi-carrier power control problem,” *Submitted to IEEE Transactions on Signal Processing*, January 2011.

[9] P. Golden, H. Dedieu, and K. Jacobsen, Eds., *Fundamentals of DSL technology*. Auerbach Publications, 2006.

[10] R. Yates, “A framework for uplink power control in cellular radio systems,” *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 7, pp. 1341–1347, September 1995.

[11] T. Morin and R. Marsten, “Branch-and-bound strategies for dynamic programming,” *Operations Research*, vol. 24, no. 4, pp. 611–627, July-August 1976.

[12] K. Price, R. Storn, and J. Lampinen, *Differential Evolution*. Springer, 2005.

[13] P. Tsiaflakis, J. Vangorp, M. Moonen, and J. Verlinden, “A low complexity optimal spectrum balancing algorithm for digital subscriber lines,” *Signal Processing*, vol. 87, no. 7, pp. 1735–1753, July 2007.

[14] F. Glover and G. Kochenberger, Eds., *Handbook of Metaheuristics*. Kluwer Academic Publishers, 2003.

[15] D. Bertsekas, “Rollout algorithms for discrete optimization: A survey,” in *Handbook of Combinatorial Optimization*, D. Zu and P. Pardalos, Eds. Springer, August 2010, to appear.

[16] M. Dorigo and L. Gambardella, “Ant colony system: A cooperative learning approach to the traveling salesman problem,” *IEEE Transactions on Evolutionary Computation*, vol. 1, no. 1, pp. 53–66, April 1997.

[17] P. Belotti, “Couenne: A user’s manual,” Lehigh University, Tech. Rep., 2009.

[18] (2008, October) xDSL simulator v3.1. [Online]. Available: xdsl.ftw.at

[19] M. Wolkerstorfer and T. Nordström, “Coverage optimization in DSL networks by low-complexity discrete spectrum balancing,” submitted to *IEEE GLOBECOM 2011*, Houston, Texas, USA, 5–9 December 2011.

[20] J. Lee, R. Sonalkar, and J. Cioffi, “Multi-user discrete bit-loading for DMT-based DSL systems,” in *IEEE GLOBECOM 2002*, vol. 2, Taipei, Taiwan, China, 17–21 November 2002, pp. 1259–1263.